MAT 1322
Lecture 17

20/03/2017

Functions in 2 variables

- $y = f(x)$ is a function in one variable
- now: $z = f(x,y)$ is a function in 2 variables
  - $\rightarrow f(x,y)$ associates a z-value to every pair of (x,y)
  - (z-value is often called the height)

Ex 1) $f(x,y) = \sqrt{x^2 - y^2}$

2) $f(x,y) = 6 - 3x - 2y$ (a plane)

3) $f(x,y) = \sin(x) \cdot \cos(y)$

Limits and continuity

In 3D there are ∞ ways to approach the top (or any point)

Ex 1) $f(x,y) = \dfrac{x^2 - y^2}{x^2 + y^2}$ What does a limit look like for $f(x,y) \rightarrow (0,0)$

<u>Domain</u>

All real tuples (pairs of #s) are allowed except $x=0$ AND $y=0$.

When we study the limit of $(x,y) \rightarrow (a,b)$ in 3D, we need to choose a direction to approach the point from.

- There are 2 natural directions:
  - along the x-axis $(y=0)$
  - along the y-axis $(x=0)$

FIRST: choose $y=0$:

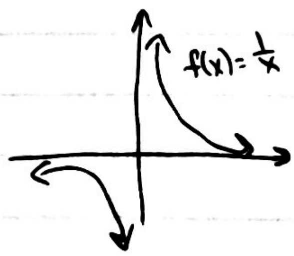
$$\lim_{(x,y)\to(0,0)} \frac{x^2 - 0^2}{x^2 + 0^2} = 1$$

now, $x=0$

$$\lim_{(x,y)\to(0,0)} \frac{\cancel{x}\, 0^2 - y^2}{0^2 + y^2} = -1$$

Different approaches give different limits.

<u>Recall</u> Calc I in 2D:



$$\lim_{x\to 0^-} f(x) = -\infty$$
$$\lim_{x\to 0^+} f(x) = \infty$$
$\ne$

In 3D:

The limit of a function $f(x,y)$ exists only if the limit from all directions exist and are the same.

If we find at least 2 directions where the limits are different, then we can say for sure the limit $\lim\limits_{(x,y)\to(a,b)} f(x,y)$ DNE at (a,b).

<u>Ex 2</u> $f(x,y) = \dfrac{x^2y^2}{x^4+3y^4}$

Domain: only $(0,0)$ isn't allowed.

$D_f = \mathbb{R}^2 \setminus \{(0,0)\}$

Step 1: go along x and y-axis first

(1) $y=0$; along x-axis

$$\lim_{(x,y)\to(0,0)} \frac{0}{x^4+3(0)^4} = 0$$

(2) $x=0$; along y-axis

$$\lim_{(x,y)\to(0,0)} \frac{0^2\cdot y^2}{0^4+3y^4} = 0$$

Step 2: check along a diagonal!

↳ Instead of $x=0$ or $y=0$ we choose $x=y$

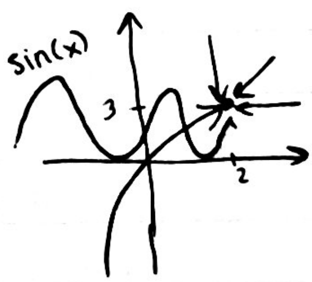
$$\lim_{(x,y)\to(0,0)} \frac{x^2\cdot x^2}{x^4+3x^4} = \lim_{(x,y)\to(0,0)} \frac{\cancel{x^4}}{4\cancel{x^4}} = \boxed{\frac{1}{4}}$$

∴ the limit of $f(x,y)$ when $(x,y)\to(0,0)$ DNE.

Other directions to approach $(x,y)\to(2,3)$:

- set $x=2$, go along y-axis
- set $y=3$, go along x-axis
- diagonally
- on a parabola

⋮



NOTE: $\mathbb{R}^2 = \mathbb{R}\times\mathbb{R}$, or the (x,y) plane here

PARTIAL DERIVATIVES

Ex 1) $f(x,y) = \underline{x}^4 \cdot y^3 + \underline{8x}^2 y$

$f_x = y^3(\underline{4x}^3) + 8y(\underline{2x})$

derive for x : leave y constant (treat as a #) and derive w.r.t. x

derive for y : leave x as constant

$f_y = x^4 \cdot (3y^2) + 8x^2(1)$

Definition of a partial derivative

w.r.t. x : $f_x(a,b) = \lim_{h \to 0} \dfrac{f(a+h,b) - f(a,b)}{h}$

w.r.t. y : $f_y(a,b) = \lim_{h \to 0} \dfrac{f(a,b+h) - f(a,b)}{h}$

'partial derivatives of f with respect to x or y at (a,b).'